A CASE STUDY OF BAYESIAN INFERENCE USING ARA 1 MODEL : APPLIED TO A RAILWAY DEVICE

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PROJECT: ZONE ÉLÉMENTAIRE DE MAINTENANCE (ZEM)

Regroup all the railway infrastructures, in geographical areas to maintain, in order to:

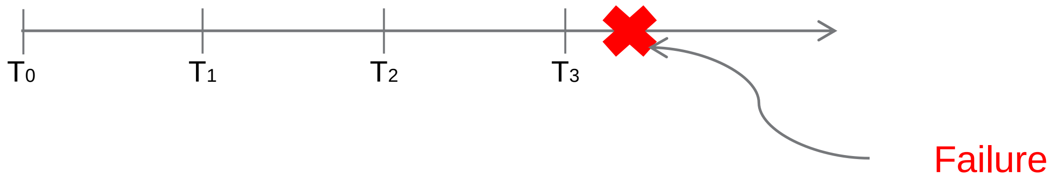
Adapt the maintenance schedules to the local need

A better availability of the railway network
Reduced maintenance costs

LOCAL APPLICATION

Study of the behavior of a signaling equipment (the target) on a particular line of FRN (line Lambda):

- Target :



2 indicators :

- Failure intensity (λ) ;
- Maintenance efficiency (ρ).

Characterristics of line lambda :

speed < 120 km/h ;

Daily traffic 30 et 40 trains.

ISSUE ?

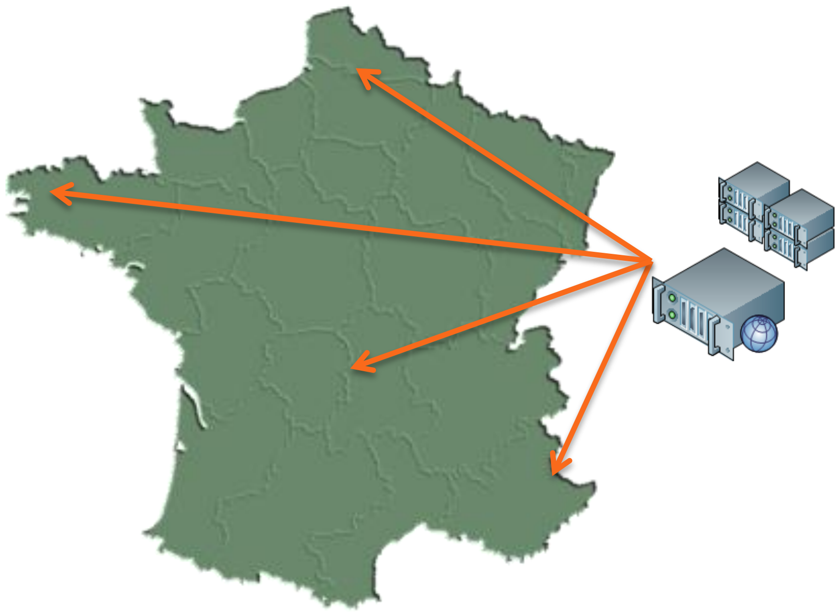
Little experience feedback data on line Lambda:

Line Lambda	failures	Censored data
target	6	22

Need to interview the maintenance personnel of this line.

METHODE (3 STEP)

1) Modelling the probability density function (national Exp Feedback) ;



2) Characteristics of the target (prior information);

3) Characteristics of the target of the line lambda (Expert reviews and Bayesian inference).

MODELLING

Impact of the maintenance: Arithmetic Reduction of Age (ARA 1)

Y: rv of the time to the first failure without preventive maintenance (PM) ;

D: rv of the time to the first failure with preventive maintenance (PM) ;

$\forall t \geq 0, N_t$: <u>deterministic</u> number of PM before time *t* ;

Conditional probability of surviving up to the j^th maintenance time T_j :

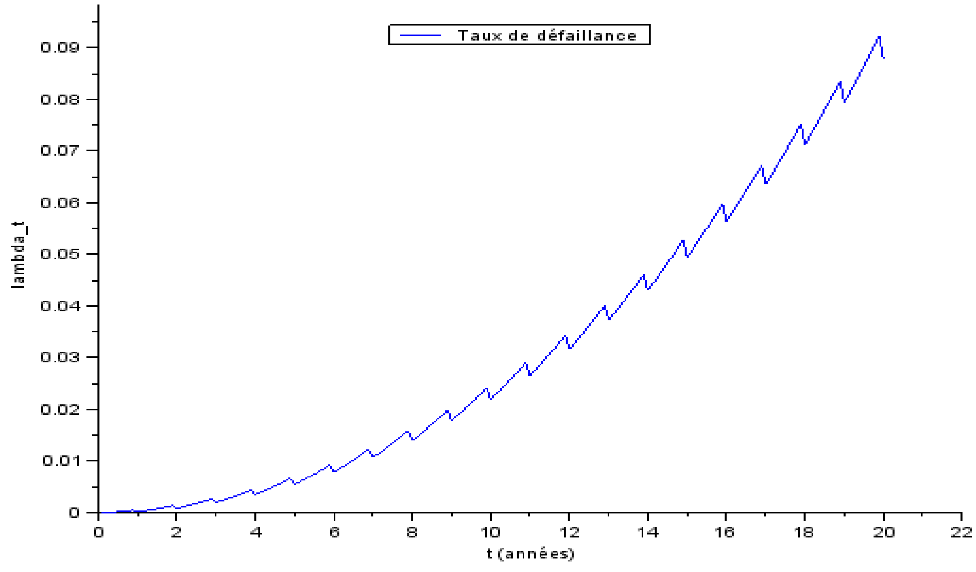
$$P(D > t \mid D > T_0, T_1, \dots, T_{Nt}) = P(Y > A_{Nt} + t - T_{Nt} \mid Y > A_{Nt})$$

A_j: Virtual age of the device after j maintenance operations → $A_j = (1 - \rho)T_j$

- ρ: impact of the maintenance → $\rho \in [0, 1]$
- Intensity of failure : $\lambda_t = \lambda(t - \rho T_{Nt})$

Weibull failure rate :

$$\lambda(u) = \frac{\beta}{\eta}\left(\frac{u}{\eta}\right)^{\beta-1}$$



BAYESIAN ESTIMATION

$R_{t_i}^{i,\theta}$	probability of surviving up to time t_i with parameter θ for the i[th] material
δ_i	$(\delta_i = 0)$ if the i[th] event is a failure, $(\delta_i = 1)$ if it is not
$\widehat{\theta_{MV}}$	ML estimator for parameter θ
$\widehat{\theta^{\pi}}$	bayesian estimator of θ with prior density function $\pi(\theta)$
H_t	history of the events occurring before time t
$\pi(\theta\|t, H_t)$	posterior density function of θ

$$\pi(\theta|t, H_t) = \frac{L(\theta|t, H_t) \times \pi(\theta)}{\int_{\Omega(\theta)} L(\theta|t, H_t) \times \pi(\theta)\, d\theta}$$

$$L\left(\theta|t_i, T_1^i, T_2^i, \dots, T_{N_{t_i}}^i, i \in \{1, \dots, n\}\right) = \prod_{i=1}^{n} [\lambda_{t_i}^{i,\theta}]^{1-\delta_i} R_{t_i}^{i,\theta}$$

$$R_{t_i}^{i,\theta} = P(D > t_i) = exp\left(-\sum_{k=0}^{N_{t_i}-1}\left(\int_{T_k}^{T_{k+1}} \lambda_u du\right) - \int_{T_{N_{t_i}}}^{t_i} \lambda_u\, du\right)$$

- **Likelihood for n materials :**

$$L\left(\beta, \eta, \rho|t_i, T_1^i, T_2^i, \dots, T_{N_{t_i}}^i, i \in \{1, \dots, n\}\right) =$$

$$\prod_{i=1}^{n}\left[\frac{\beta}{\eta}\left(\frac{t_i - \rho T_{N_{t_i}}^i}{\eta}\right)^{\beta-1}\right]^{1-\delta_i} exp\left(-\sum_{k=0}^{N_{t_i}-1}\left[\left(\frac{T_{k+1}^i - \rho T_k^i}{\eta}\right)^{\beta} - \left(\frac{T_k^i - \rho T}{\eta}\right)^{\beta}\right]\right) exp\left(-\left[\left(\frac{t_i - \rho T_{N_{t_i}}^i}{\eta}\right)^{\beta} - \left(\frac{T_{N_{t_i}}^i - \rho T_{N_{t_i}}^i}{\eta}\right)^{\beta}\right]\right)$$

- **Uniform prior density :**

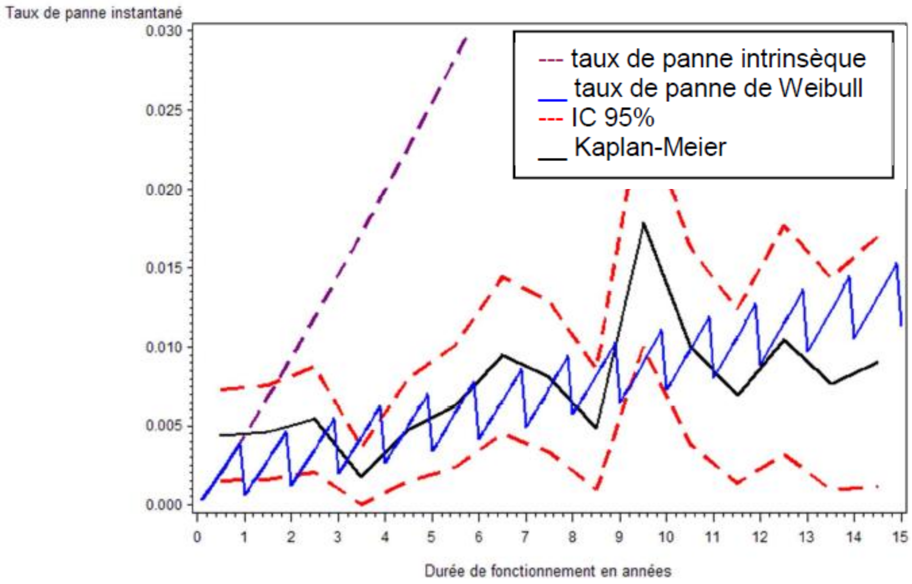
$$\pi(\theta) = \frac{1}{\theta_u - \theta_l} 1[\theta_l, \theta_u](\theta)$$

STUDY OF THE TARGET

Step 1: national Exp Feedback

Inference (ML):

Failures	Censored data	$\widehat{\beta_{MV}}$	$\widehat{\eta_{MV}}$	$\widehat{\rho_{MV}}$
936	1928	2.1	19	0.84



Step 2 : Characteristics of the target

- β : Uniform β∈[1,4] (the device is ageing)
- η : Uniform, centred on 19
- Parameter maintenance efficiency, estimated by the experts:

Strong impact of the PM : ρ∈[0.7 , 1]
Medium impact of the PM : ρ∈[0.4 , 0.7]
Week impact of the PM : ρ∈[0.1 , 0.4]

STUDY OF THE TARGET

Step 3: Characteristics of line Lambda

	Target			
Parameters	ML inference (National)	Uniform prior	Prior expectation	Posterior estimator (Line Lambda)
β	$\widehat{\beta_{MV}} = 2.1$	[1, 4]	2.5	$\widehat{\beta^{\pi}} = 1.34$
η	$\widehat{\eta_{MV}} = 19$	[14, 24]	19	$\widehat{\eta^{\pi}} = 19.34$
ρ	$\widehat{\rho_{MV}} = 0.84$	[0.7, 1]	0.85	$\widehat{\rho^{\pi}} = 0.84$
				6 failures & 22 censored data

Comments :

Concordance between expert knowledge and ExpFeedback to the impact of preventive maintenance;

Behavior of target on line Lambda ≠ Behavior of target on the FRN :

same impact of maintenance but they age slower.

specific operating conditions on line Lambda → suggests an adaptation of the maintenance program of the line.

ANALYSE OF THE RESULTS

Simulate K = 5000 samples n ϵ {5, 50, 100} failures, wherein: $(\beta, \eta, \rho) = (2, 19, 0.8)$

Bayesian estimation using Monte Carlo integrals on each prior interval.

Prior interval of scale and shape parameters:

$$\eta \in [14, 24], \beta \in [1, 4]$$

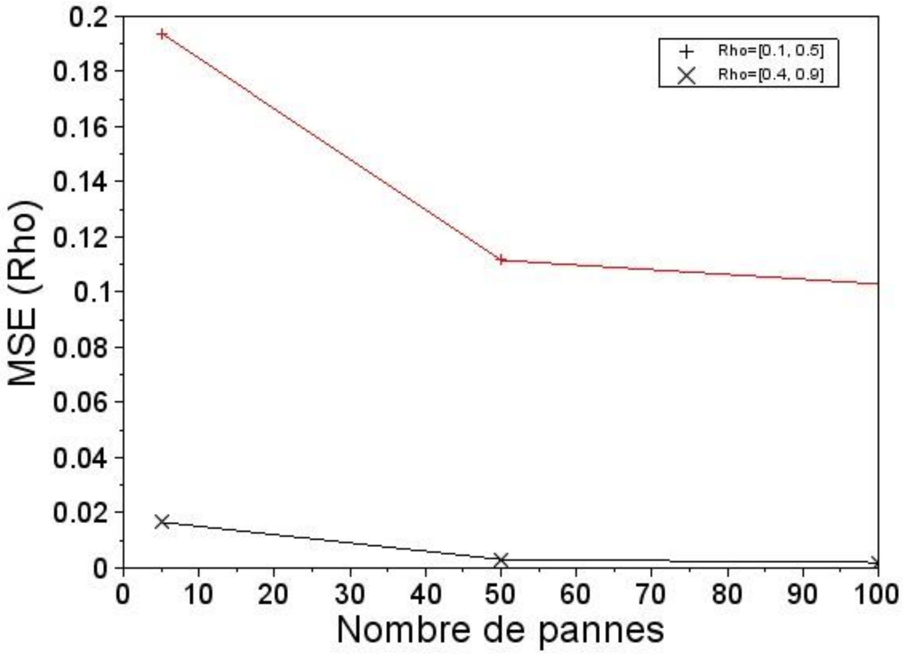
Prior interval of the impact of preventive maintenance:

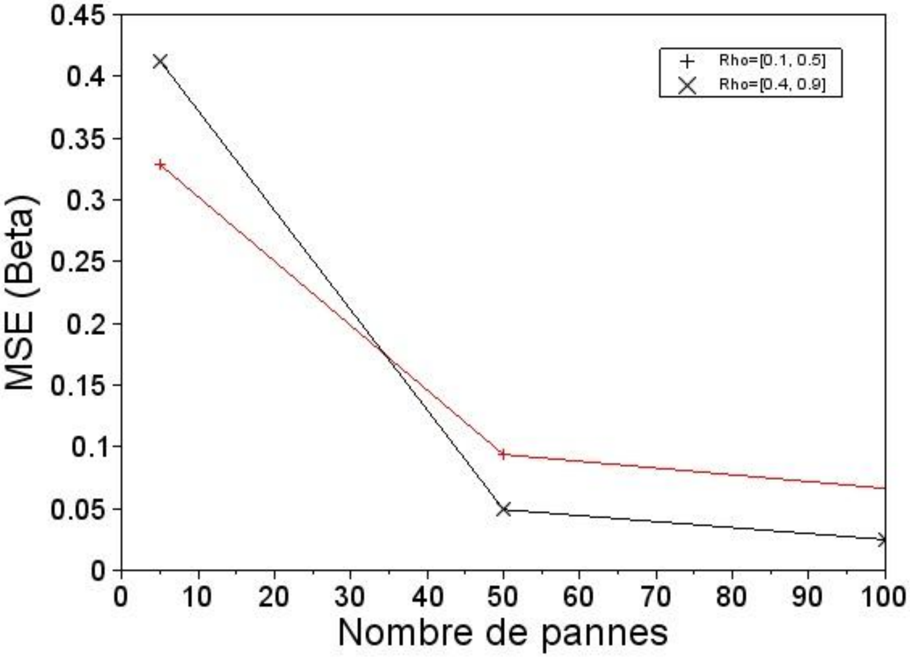
- **Interval containing the true value (0.8): ρϵ[0.4 , 1]**
- **Interval that does not contain the true value: ρϵ[0.1 , 0.5]**

Sensitivity of the results is obtained by estimating the MSE criterion:

$$\widehat{MSE(\hat{\theta})} = S^2(\hat{\theta}) + (Moy(\hat{\theta}) - \theta)^2$$

ANALYSE OF THE RESULTS





ANALYSE OF THE RESULTS

$\rho \in [0.4\,, 0.9]$	$Moy(\widehat{\beta})$	$Moy(\widehat{\rho})$
n = 5	2.4069112	0.6914922
n = 50	2.0494821	0.7594486
n = 100	2.022743	0.7677337

$$\beta(real) = 2\,; Prior\ expectation\ (\beta) = 2.5$$
$$\rho(real) = 0.8\,; Prior\ expectation(\rho) = 0.65$$

$\rho \in [0.1\,, 0.5]$	$Moy(\widehat{\beta})$	$Moy(\widehat{\rho})$
n = 5	1.9625624	0.3609510
n = 50	1.8210478	0.466256
n = 100	1.8269931	0.4792167

$$\beta(real) = 2\,; Prior\ expectation(\beta) = 2.5$$
$$\rho(real) = 0.8\,; Prior\ expectation(\rho) = 0.3$$

CONCLUSION

The Method of inference :

- Interest of relying on three sources of information (National network + local failures + local expert knowledge);
- Some very explicit parameters (η) does not need to be determined by the experts;
- Rely on the knowledge of the maintenance personnel ;
- impact of potential bias introduced by the expert (due to little ExpF on line Lambda);

THANK YOU !

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